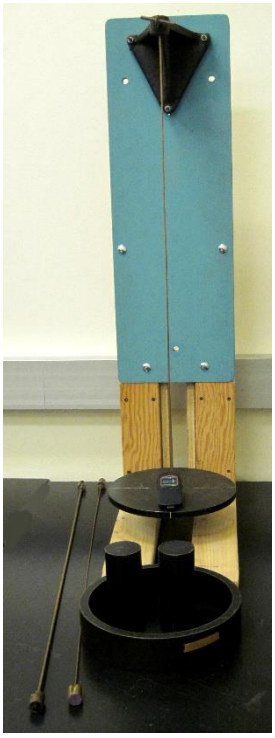
Torsion Pendulum

Component Properties:



Brass Rods:

R_{rod1} = 0.00116 m
R_{rod2} = 0.00156 m
R_{rod3} = 0.00236 m

$\mathscr{l}$ = 0.865 m

Disk:

R_{disk} = 0.1270 m
M_{disk} = 4.56 kg

$$I_{disk} = {}^{1}/_{2}\, M_{disk} R_{disk}^2 = 1/2(4.56)(0.1270)^2$$
$$= 0.0368 \text{ kg-m}^2$$

Ring:

$R_{ring:in}$ = 0.1126 m
$R_{ring:out}$ = 0.1270 m
M_{ring} = 4.30 kg

$$I_{ring} = {}^{1}/_{2}\, M_{ring}\left(R_{ring-in}^2 + R_{ring-out}^2\right)$$
$$= 1/2(4.30)(0.1126^2 + 0.1270^2)$$
$$= 0.0619 \text{ kg-m}^2$$

Cylinder:

R_{cyl} = 0.0285 m
M_{cyl} = 2.67 kg
d = 0.0985 m

Around the Center of Mass:

$$I_{cyl:CM} = {}^{1}/_{2}\, M_{cyl} R_{cyl}^2$$
$$= 1/2(4.30)(0.1126^2 + 0.1270^2)$$
$$= 0.0619 \text{ kg-m}^2$$

Combinations of Components:

Disk with Ring:

$$I_{disk+ring} = I_{disk} + I_{ring}$$
$$= 0.0368 + 0.0619$$
$$= 0.0987 \text{ kg-m}^2$$

Disk with Two Cylinders at opposite edges of the disk:

d = distance between center of disk and center of cylinder = 0.0985 m

Parallel axis theorem:

$$I_{cyl} = I_{cyl:CM} + M_{cyl}d^2$$

For two cylinders:

$$I_{2cyl} = M_{cyl}\left(R_{cyl}^2 + 2d^2\right)$$
$$= (2.67)[(0.0285)^2 + 2(0.0985)^2]$$
$$= 0.0540 \text{ kg-m}^2$$

$$I_{disk+2cyl} = I_{disk} + I_{2cyl}$$
$$= 0.0368 + 0.0540$$
$$= 0.0908 \text{ kg-m}^2$$

Torsion Constant of the Rod κ

Modulus of Rigidity for Brass $n = 40\times10^9$ kg/m-s^2

$$\kappa_{rod1} = \frac{n\pi R_{rod1}^4}{2\ell} = \frac{(40\times10^9)\pi(0.00118)^4}{2(0.865)} = 0.131 \text{ N-m/rad}$$

$$\kappa_{rod2} = \frac{n\pi R_{rod2}^4}{2\ell} = \frac{(40\times10^9)\pi(0.00156)^4}{2(0.865)} = 0.426 \text{ N-m/rad}$$

$$\kappa_{rod3} = \frac{n\pi R_{rod3}^4}{2\ell} = \frac{(40\times10^9)\pi(0.00236)^4}{2(0.865)} = 2.23 \text{ N-m/rad}$$

Calculated Periods using Rod #1:

Disk: $T_{disk} = 2\pi\sqrt{\frac{I_{disk}}{\kappa_{rod1}}}$ = 3.22 s

Disk + Ring: $T_{disk+ring} = 2\pi\sqrt{\frac{I_{disk+ring}}{\kappa_{rod1}}}$ = 5.45 s

Disk + 2 Cylinders: $T_{disk+2cyl} = 2\pi\sqrt{\frac{I_{disk+2cyl}}{\kappa_{rod1}}}$ = 5.23 s

Measured Periods using Rod #1:

Disk: T_{disk} = 3.40 s

Disk + Ring: $T_{disk+ring}$ = 5.55 s

Disk + 2 Cylinders: $T_{disk+2cyl}$ = 5.36 s

Calculated Periods using Rod #2:

Disk: $T_{disk} = 2\pi\sqrt{\frac{I_{disk}}{\kappa_{rod2}}}$ = 1.85 s

Disk + Ring: $T_{disk+ring} = 2\pi\sqrt{\frac{I_{disk+ring}}{\kappa_{rod2}}}$ = 3.02 s

Disk + 2 Cylinders: $T_{disk+2cyl} = 2\pi\sqrt{\frac{I_{disk+2cyl}}{\kappa_{rod2}}}$ = 2.90 s

Measured Periods using Rod #2:

Disk: T_{disk} = 1.89 s

Disk + Ring: $T_{disk+ring}$ = 3.06 s

Disk + 2 Cylinders: $T_{disk+2cyl}$ = 2.95 s

Calculated Periods using Rod #3:

Disk: $T_{disk} = 2\pi\sqrt{\frac{I_{disk}}{\kappa_{rod3}}}$ = 0.81 s

Disk + Ring: $T_{disk+ring} = 2\pi\sqrt{\frac{I_{disk+ring}}{\kappa_{rod3}}}$ = 1.32 s

Disk + 2 Cylinders: $T_{disk+2cyl} = 2\pi\sqrt{\frac{I_{disk+2cyl}}{\kappa_{rod3}}}$ = 1.27 s

Measured Periods using Rod #3:

Disk: T_{disk} = 0.83 s

Disk + Ring: $T_{disk+ring}$ = 1.34 s

Disk + 2 Cylinders: $T_{disk+2cyl}$ = 1.29 s